Loan Management under Credit Risk and Future Capitalization

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A two-period option model of a bank was used to determine the optimal lending under future capitalization is developed. The optimal lending on the market value of bank equity under future expandability or reversibility is that the first-period marginal equity value of lending is equal to the second-period marginal equity value. A principal finding supports a conclusion for securing of bank loans that an increase in volatility may make the bank become conservative. This conservative lending may be influenced by the expected decreasing resale price and by the expected increasing purchase price of future equity capital.

1. Introduction

It has been recognized that the asset growth of banks has significantly outpaced the growth rate of their capital in the last few years. Increasing risks in the bank lending can be perceived since primary capital-to-asset ratios have declined. Recognizing the importance of asset management, banks have sought many ways to manage the associated risks. As pointed out by Hendricks & Hirtle (1997), many banks have taken serious steps to ensure that their management has adequate internal controls and capital resources to address these risks. Nagle & Peterson (1985) demonstrated that bank equity capital is a source to absorb funds and is a cushion to absorb unexpected operating losses as well. The relationship between capital and assets (e.g. lending) is then critical for analyzing bank management. Syron (1991) presents the so-called credit-crunch relationship, which links a bank's ability to make loans to the amount of capital it has against loan losses. According to Bernanke & Lown's (1991) study, there is a causal link between low capital-to-asset ratios and lending growth in the subsequent recession. In consequence, a credit crunch is expected to occur in the subsequent recession. We further argue that the so-called "reversed" credit crunch may be expected in the subsequent expansion. As pointed out by Abel, Dixit, Eberly & Pindyck (1996), when a bank cannot costlessly adjust its equity capital, it must consider future opportunities and costs when making its investment decisions.

In light of previous work, the purpose of this paper is to develop a model that integrates the optimal lending (internal controls) with changes in reversibility or expandability of future capital (capital resources in Hendricks & Hritle's sense).[1] The results of this paper show how current/future volatilities, and resale/purchase price of future capital jointly determine the optimal bank loan. We find that the bank's optimal lending is a decreasing function of its current volatility under the rebalancing effect and has no effect caused by its future volatility under the gamma neutral in the sense of Hull (1993). In addition, the optimal lending is an increasing (decreasing) function of the resale (purchase) price of future capital if the intertemporal lending with reversibility (expandability) is greater than (less than and equal to) the one-plus risk-free rate.

Subsequent sections present the basic framework for this analysis. We used a simple, two-period framework that incorporates only the necessary features: first-and second-period returns are stochastic (uncertainty). Sections III is devoted to the analysis of comparative statics, including volatilities in different periods and future resale and purchase prices. Section IV contains the concluding remarks.

11. The Model

We analyze the bank's lending decisions associated with the possible later selling or expanding capacity. Following Abel, Dixit, Eberly & Pindyck, the possible later selling capacity demonstrates the bank's future disinvestment, but the resale price of capacity may be less than its current acquisition price (called costly reversibility)[2]. Similarly, the possibility of later buying capacity demonstrates the bank future investment, but the future acquisition price of capacity may be higher than its current acquisition price (called costly expandability). Either a costly reversion or a costly expansion in future investment may affect the bank's current decisions for lending. To capture the features of future capitalization, we set up a simple, two-period framework as follows. Consider a bank with no legal reserve requirement facing uncertain loan repayments due to credit risk and later selling/expanding its capacity. At the starting point of the first-period horizon, the bank raises D_0 in deposits and E_0 in equity. The balance sheet is presented using $V_0 = D_0 + E_0$ (1)

where VO is the value of the bank's loans. As pointed out by Mullins & Pyle (1994), the equity can be expressed as the Black-Scholes (1973) value of a call option purchase made effectively by the bank's shareholders. This call option is written on the bank's assets with uncertainty expressed by its standard deviation of the return and written with an exercise price equal to the promised payments to the depositors.

While we adopt no restrictions on the detailed characteristics of the deposits,[3] such as random net deposit flow, it may be assumed that the initial deposits D_0 mature at the end of the second-period horizon if neither reversion nor expansion occurs. But if this is not the case, D_0 is assumed to mature at the end of the first-period horizon. In the second-period horizon, after the reversion or expansion strategy is conducted, the bank raises D_{1b} in deposits and E_{1b} in equity. Furthermore, we assume that D_{1b} matures at the end of the second-period horizon. We assume that an audit is executed whether or not reversion or expansion is performed at the end of each-period horizon, but the audit cost is assumed to be equal to zero.[4] Following Mullins & Pyle, the bank's first-period equity value is shown using

$$E_1 (V_0, D_1, \sigma_1) = V_0 N(d_1) - D_0 e^{-2\mu_0} N(d_1 - \sigma_1 \sqrt{2}) \quad (2)$$

where,

$$d_1 = \frac{\ln(V_0/D_0) + 2\mu_0}{\sigma_1\sqrt{2}} + \frac{\sigma_1}{\sqrt{2}}$$

$$\mu_0 = r_f - r_D$$

σ_1 is the standard deviation for the return on the bank's assets during the first period. N(·) is the cumulative standard normal distribution. μ_0 is the deposit rate spread; i.e., the spread between the risk-free rates (r_f) and the promised interest rate (r_D) to the initial deposit. For purposes of simplicity, assume that this deposit rate spread remains constant during the two periods of the model.

Without conducting the reversion or expansion activity, the bank's second-period equity value is shown using

$$E_2(V_1(V_0, \sigma_1), D_2, \sigma_2) = V_1(V_0, \sigma_1)N(d_2) - D_1e^{-2\mu_0}N(d_2 - \sigma_2\sqrt{2}) \quad (3)$$

where,

$$d_2 = \frac{\ln(V_1(V_0, \sigma_1)/D_1) + 2\mu_0}{\sigma_2\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}}$$

σ_2 is the standard deviation for the return on the bank's assets during the second-period horizon if either reversion or expansion activity is not conducted. D_1 is the initial deposit value at the starting point of the second-period horizon. The value of the second-period bank's assets also depends on the first-period assets as well as its standard deviation. There are two crucial factors shown in equation (3): delta hedging ($\delta E_2 / \delta V_1 \geq 0$) and intertemporal lending ($\delta V_1 / \delta V_0$). Applying Hull (1993, p.299), the delta hedging factor is positive and explain the (reversed) credit-crunch effect of the rate of change the bank's equity with respect to its lending. The intertemporal lending factor states the interaction of the associated two-period lending businesses. Based on a rather general assumption, it is reasonable to believe that the first-period loan and the second-period loan have the nature of complements since they are the same product appearing in the different periods. Thus, $\delta V_1 / \delta V_0 > 0$. Under this assumption, the value of equity during the two-period horizon without reversibility or expandability activity is demonstrated by [5]

$$M(V_0) = \delta E_2(V_1(V_0, \sigma_1), D_2, \sigma_2) \quad (4)$$

where the discounted factor δ is positive and less than one.

The bank's second-period equity value under conducting the reversion or expansion activity without considering the revenue from reversion or the cost of expansion is shown using

$$E_{2b}(V_{1b}(V_0, \sigma_1), D_{2b}, \sigma_2) = V_{1b}(V_0, \sigma_1)N(d_{2b}) - D_{1b}e^{-2\mu_0}N(d_{2b} - \sigma_2\sqrt{2}) \quad (5)$$

where,

$$d_{2b} = \frac{\ln(V_{1b}(V_0, \sigma_1)/D_{1b}) + 2\mu_0}{\sigma_2\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}}$$

The interpretation of $\delta E_{2b}/\delta V_{1b}$ (delta hedging with reversion or expansion) and $\delta V_{1b}/\delta V_0$ (intertemporal lending with reversion or expansion) follows a similar argument as in the case of without reversion or expansion. Given the above assumption, the bank's second-period equity value under the reversion regime is

$$M_r(V_0) = \delta[E_{2b}(V_{1b}(V_0, \sigma_1), D_{2b}, \sigma_2) + P_L((1+r_f)E_0 - E_{1b})] \tag{6}$$

where $P_L((1+r_f)E_0 - E_{1b})$ is the revenue from selling the part of the bank's equity at the starting point of the second period. P_L is the resale price of equity which is treated as the marginal return of $E_{2b}(V_{1b}(V_0, \sigma_1), D_{2b}, \sigma_{2L})$. When $P_L < 1$, the resale price of equity is less than its current (period 1) price and we have reversion costs for investment. Symmetrically, the case for the expansion regimen is given using

$$M_e(V_0) = \delta[E_{2b}(V_{1b}(V_0, \sigma_1), D_{2b}, \sigma_2) + P_H(E_{1b} - (1+r_f)E_0)] \tag{7}$$

where $P_H(E_{1b} - (1+r_f)E_0)$ is the cost of purchasing the bank's extra equity at the starting point of the second period. P_H is the purchase price of equity and we assume that this price is equal to the marginal return of $E_{2b}(V_{1b}(V_0, \sigma_1), D_{2b}, \sigma_{2H})$. When $P_H > 1$, the purchase price of equity is greater than its current (period 1) price and we have costly expansion for investment.

Investment decisions at the starting point of the second-period horizon are made under three possible regimes since σ_2 may be less than σ_{2L}, between σ_{2L} and σ_{2H}, or greater than σ_{2H}. Thus, the bank's initial lending decision problem is

$$\underset{V_0}{\text{Max}}\, J = \left[\int_{-\infty}^{\sigma_{2L}} M_r(V_0)\delta F(\sigma_2) + \int_{\sigma_{2L}}^{\sigma_{2H}} M(V_0)\delta F(\sigma_2) + \int_{\sigma_{2H}}^{\infty} M_e(V_0)\delta F(\sigma_2)\right] - E_0 \tag{8}$$

The first-order condition for this maximization is given using

$$\delta\left[\frac{\delta E_{2b}}{\delta V_{1b}}\frac{\delta V_{1b}}{\delta V_0}(1-F(\sigma_{2H})) + ((1+r_f) - \frac{\delta V_{1b}}{\delta V_0}(P_H(1-F(\sigma_{2H})) + P_L F(\sigma_{2L})) + \int_{\sigma_{2L}}^{\sigma_{2H}} \frac{\delta E_{2b}}{\delta E_{2b}}\frac{\delta V_{1b}}{\delta V_{1b}} dF(\sigma_2)\right] = 1 \tag{9}$$

The left side of equation (9) is viewed as the discount second-period marginal equity value of lending. The first term of the marginal value is the second-period marginal value related to V_0 and V_{1b} if either reversion or expansion takes place (net reversibility effect). The second term is related to the resale revenue or the purchase cost (net revenue effect). The third term is the value representing the case of neither reversion nor expansion (constant capitalization effect). The right side of equation (9) is viewed as the first-period marginal equity value of lending; that is the unit price of equity (equals/in this model). Thus, the equilibrium states the optimal lending decision should equate both periods' marginal equity value of lending. Note that both the delta hedging and intertemporal lending factors have influences on the net reversion effect as well as the constant capitalization effect. The intertemporal lending factor influences the net revenue effect. As pointed out by Hull (1993, p.299), a zero delta position referred to as being delta neutral states that the delta of the asset position offsets the option position. This is the so-called rebalancing effect when both the net reversion and the constant capitalization effects are equal to zero due to delta neutral. We find that the net revenue effect shown by the intertemporal lending behavior completely dominates the optimal lending decision under the rebalancing interaction. Our finding in the equilibrium condition follows Hendricks & Hirtle's argument of risk management emphasizing internal controls (intertemporal lending) and capital resources (resale revenue or purchase cost in this paper).

111. Comparative Statics

Consider the impact on the bank's lending from changes in volatility during first-period and second-period horizons, and resale/purchase price of equity.

Implicit differentiation of equation (9) with respect to σ_1 yields

$$\frac{\delta V_0}{\delta \sigma_1} = \frac{H_{\sigma 1} + G_{\sigma 1} + K_{\sigma 1}}{\delta^2 J / \delta V_0^2} \tag{10}$$

where,

$$H_{\sigma 1} = \frac{\delta^2 E_{2b}}{\delta V_{1b}^2} \frac{\delta E_{2b}}{\delta V_{1b}} \frac{\delta V_{1b}}{\delta \sigma_1} + \frac{\delta E_{2b}}{\delta V_{1b}} \frac{\delta^2 V_{1b}}{\delta V_0^2}) \frac{\delta V_{1b}}{\delta V_0} (1\text{-}F(\sigma_{2H})+F(\sigma_{2L}))$$

$$G_{\sigma 1} = \frac{\delta^2 V_{1b}}{\delta V_0^2} \frac{\delta V_{1b}}{\delta \sigma_1} (P_H(1\text{-}F(\sigma_{2H}))+P_L F(\sigma_{2L}))$$

$$K_{\sigma 1} = \int_{\sigma_{2L}}^{\sigma_{2H}} (\frac{\delta^2 E_2}{\delta V_1^2} \frac{\delta E_2}{\delta V_1} \frac{\delta V_1}{\delta \sigma_1} + \frac{\delta E_2}{\delta V_1} \frac{\delta^2 V_1}{\delta V_0^2}) \frac{\delta V_1}{\delta V_0} dF(\sigma_2)$$

We assume that $\delta^2 J / \delta V_0^2 < 0$ since the second-order condition must be satisfied. $H_{\sigma 1}$ is the marginal net reversion effect considering changes in the second-period lending with the first-period volatility. Similarly, $G_{\sigma 1}$ is the marginal net revenue effect and $K_{\sigma 1}$ is the marginal constant capitalization effect. Equation (10) indicates that increase in the first-period volatility decreases the bank's lending under the rebalancing effect in Hull's sense. This in because the net revenue effect completely dominates the optimal lending decision. $G_{\sigma 1} > 0$ since a risk-averse bank's marginal adjustment of intertemporal lending behavior is generally observed to be conservative. This observation allows us make such an assumption of $\delta^2 V_{1b} / \delta V_0^2 < 0$. However, an increase in the first-period volatility has an indeterminate effect on the bank's optimal lending if the rebalancing effect is not shown.

Implicit differentiation of equation (9) with respect to σ_2 yields

$$\frac{\delta V_0}{\delta \sigma_2} = \frac{H_{\sigma 2} + K_{\sigma 2}}{\delta^2 J / \delta V_0^2} \qquad (10)$$

where,

$$H_{\sigma 2} = \frac{\delta^2 E_{2b}}{\delta V_{1b}^2} \frac{\delta E_{2b}}{\delta \sigma_2} \frac{\delta V_{1b}}{\delta V_0^2} (1-F(\sigma_{2H})+F(\sigma_{2L}))$$

$$K_{\sigma 1} = \int_{\sigma_{2L}}^{\sigma_{2H}} (\frac{\delta^2 E_2}{\delta V_1^2} \frac{\delta E_2}{\delta \sigma_2} \frac{\delta V_1}{\delta V_0} dF(\sigma_2)$$

Equation (11) shows that an increase in the second-period volatility has no effect on the first-period lending under the gamma neutral in Hull's sense.[6] Besides the intertemporal effects, gamma hedging plays an important role in determining the changes for optimal lending with respect to future volatility. Applying Hull (1993), if gammas approach to zero is relatively insignificant, delta changes only slowly and adjustments to keep a lending delta neutral need only be made relatively infrequently. Thus, the second-period volatility have no effect on the bank's first-period optimal lending even though the intertemporal lending modes of behavior are considered.

Consider further the impact on the bank's lending from changes in the resale price and the purchase price of equity. Implicitly differentiating V_0 of equation (9) with respect to P_L and P_H, respectively, we obtain

$$\frac{\delta V_0}{\delta P_L} = \frac{((1+r_f) - \delta V_{1b}/\delta V_0)F\sigma_{2L}}{\delta^2 J / \delta V_0^2} \qquad (12)$$

$$\frac{\delta V_0}{\delta P_H} = \frac{((1+r_f) - \delta V_{1b}/\delta V_0)F\sigma_{2H}}{\delta^2 J / \delta V_0^2} \qquad (13)$$

The optimal lending increase as the resale price of capital increases if the intertemporal lending with reversion is greater than the one-plus risk-free rate $(1+r_f)$. Increasing the price at which capital can be sold in the future raises the value of option to sell capital, and thus reduces the effective cost of investment and increases the optimal lending value. Similarly, the optimal lending decreases as the future purchase price of capital decreases the intertemporal lending with expansion is less than and equal to the one-plus risk-free rate. Decreasing the price at which capital can be purchased in the future raises the value of option to buy capital and thus increases the optimal value of lending. Our findings support Abel, Dixit, Eberly & Pindyck (1996, p.764).

IV. Conclusion

Loan management under future capitalization has been the subject of extensive research recently in connection with the credit or capital crunch. This paper established a two-period option framework to study how to manage loans under costly reversion and expansion. The optimal lending decision should equate the bank's first-period marginal equity value of lending with its second-period one. We find that an increase in the first-period volatility decreases the bank's lending under the rebalancing effect in Hull's sense and an increase in the second-period volatility has no effect on the bank's lending under the gamma neutral in Hull's sense. Furthermore, the optimal lending increases as the resale (purchase) price of future capital increases (decreases) if the intertemporal lending with reversion (expansion) is greater than (less than and equal to) the one-plus risk-free rate. This paper suggests that future capitalization can also be important in influencing the bank's lending decision.

Footnotes

1 As presented by Syron (1991, p.4), "shrinking credit availability from banks may be more accurately characterized as a capital crunch rather than a credit crunch." and "tight credit is the result of a loss (decrease) in bank capital, rather than a loss in deposits." The concept of reversibility of capital in the model can be treated as a kind of capital crunch. Symmetrically, loose credit may be the result of a gain (increase) in bank capital. The concept of expandability of capital in the model can be treated as a sort of reversed capital crunch.

2 We admit that tight credit is one of the results of bank capital loss. The future disinvestment of later selling capacity, either gain or loss, may push a bank toward the tight-credit status.

3 According to Syron (1991,p.40), "...this period of tight credit is the result of a loss (decrease) in bank capital, rather than a loss in deposits." The focal point of this paper is to analyze the relationships between lending and capitalization. Therefore, the detailed characteristics of deposits are ignored.

4 Mullins & Pyle (1994) assume that total audit costs are proportional to the size of the bank's total asset portfolio. We expected that the quantitative results will not change if the assumption above is incorporated to analyze the credit crunch effect of the paper. Thus, for the sake of simpler analysis, we assume zero audit cost in order to avoid potential confusion.

5 $M(V_0)$ cannot be affirmatively treated as $E_2\,(V_0, D_2, \sigma_1 + \sigma_2)$ since the second-period return is calculated under three possible regimes: reversion, expansion or neither of them.

6 According to Hull (1993), gamma is defined as the rate of changes of the values of the portfolio with respect to delta. The gammas of the equations (10) are $\frac{\delta^2 E_{2b}}{\delta V_{1b}^{\,2}}$ and $\frac{\delta^2 E_{2b}}{\delta V_{1b}^{\,2}}$ and a zero gamma position is referred to a gamma neutral.

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